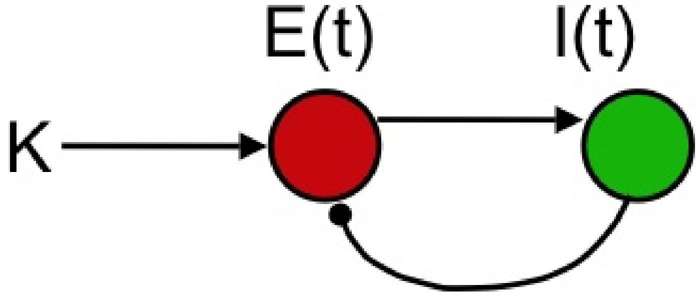
Theoretical Neuroscience II
Computational Neuroscience II
State Space Analysis
(20 points)

Please analyze the two dynamic systems described below. Both systems consist of one excitatory and one inhibitory node.



1 2D linear system (10P)

$$\frac{dE}{dt} = \frac{1}{\tau}\,(E - \alpha\, I + K)$$
$$\frac{dI}{dt} = \frac{1}{\tau}\,(-I + \beta\, E - 4\, K)$$

where $\tau = 10$, $\alpha = 2$, $\beta = 5$ and K is an external input which is either 0 or 4.

2 2D non-linear system (Wilson-Cowan oscillator) (10P)

$$\frac{dE}{dt} = \frac{1}{\tau_E}\,[-E + S(\alpha\, E - I + K)]$$
$$\frac{dI}{dt} = \frac{1}{\tau_I}\,[-I + S(\beta\, E)]$$

where $\tau_E = 5$, $\tau_I = 10$, $\alpha = 1.6$, $\beta = 1.5$ and K is an external input which is either 0 or 20. The non-linear function $S(x)$ is defined as

$$S(x) = \frac{Mx^2}{\sigma^2 + x^2}$$

with $M = 100$ and $\sigma = 30$.

3 Assignments

Your general task is to analyze the systems' behaviour with respect to stability for each of the two values which are specified for the input K! The provided Matlab scripts will support you and allow verifying your results. They assemble a map of the state space, with activity E on the x-axis and activity I on the y-axis. Also, they allow to generate single trajectories in the state space from a given start point. To apply the scripts minor completion work is necessary (compare last item).
For each value of K do the following:

- Derive the isocline equations (by setting $\frac{dE}{dt} = 0$ or $\frac{dI}{dt} = 0$).
- Calculate the intersection of the isoclines to determine the equilibrium point(s)! If you are analyzing the non-linear system, you will have to do this numerically. The provided script already has implemented this functionality, hence simply check the matlab output.
- Define auxiliary variables F_E and F_I as follows

  $$F_E \equiv \frac{dE}{dt}, \qquad F_I \equiv \frac{dI}{dt}$$

  Derive the *Jacobian*

  $$J = \begin{pmatrix} \frac{\delta F_E}{\delta E} & \frac{\delta F_E}{\delta I} \\ \frac{\delta F_I}{\delta E} & \frac{\delta F_I}{\delta I} \end{pmatrix}$$

  For the linear system, the Jacobian is the same everywhere in state space. For the non-linear system, the Jacobian depends on the position in state space and must be evaluated at the equilibrium point (which you have determined above).
- Compute the eigenvalues of the Jacobian and determine whether the system is stable.
- **Matlab:**
  - For each value of K, use the provided Matlab scripts to draw a "dynamic flow field" over your state space. In case of the non-linear system the file getS.m has to be completed before!
  - Determine the eigenvalues (matlab command: eig(matrix)) of the Jacobian of your system. Therefore, complete the file evaluateJacobianL.m in case of the linear system and evaluateJacobianNL.m in case of the non-linear system with your Jacobian matrix!
  - Further, generate example trajectories to verify your results (discrete evolution part in scripts).
  - Finally, describe the behaviour of the systems in qualitative terms, especially with respect to fixpoints!