7^{th} Exercise in Digital Information Processing

1. Matrix potention with eigen-values

   - Given is a system of order 2 with equation:

   $$y(n) = a_1 u(n-1) + a_0 u(n-2) - b_1 y(n-1) - b_0 y(n-2) \tag{1}$$

   - Sketch an equivalent block-diagram.
   - Define two state variables $x_1(n)$ and $x_2(n)$ and rewrite the system equation for $x_1(n+1)$ and $x_2(n1)$
   - Solve the system with matrix potention for the variables:
     $b_0 = -3/4$ and $b_0 = -1$

2. Autocorrelation estimation

   - Given is a system of order 2 with equation:

   $$x[n] = q[n] - \frac{1}{4}x[n-1] + \frac{1}{2}x[n-2] \tag{2}$$

   - Assume that all parameters at $t = 0$ are zero. The system is than excited with the delta function $q[n] = \delta[n]$. Please compute $x[0]$ to $x[3]$.
   - Use the consistent autocorrelation estimator

   $$\hat{S}_{XX}[|\kappa|] = \frac{1}{N} \sum_{\kappa=0}^{N-1-|\kappa|} x[n]x[n+\kappa] \tag{3}$$

     and the first 4 elements of $x[n]$ to compute $\hat{S}_{XX}[0]$ to $\hat{S}_{XX}[3]$.
   - Compute $x[4]$ to $x[6]$ and use all seven elements of $x[n]$ to compute $\hat{S}_{XX}[0]$ to $\hat{S}_{XX}[3]$ again. Compare the results to the true values of the autocorrelation function:
     $S_{XX}[0] = 0.4444, S_{XX}[1] = -0.2222,$
     $S_{XX}[2] = 0.2777, S_{XX}[3] = -0.1805$